



Australian Government

Department of Defence

Science and Technology



Coherent simulation of sea-clutter for a scanning radar

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Airborne maritime surveillance

- Detection of small targets on the sea surface
 - Currently operate airborne platforms at low altitude.
 - Low grazing angles have reduced clutter backscatter.
 - Non-coherent processing.
 - Single aperture systems.
- Desire to operate with platforms at higher altitudes.
 - UAVs flying at $> 10,000$ ft.
- Future radars will likely be multi-aperture and use coherent processing.
- Most clutter simulation techniques only consider a single look direction.
- Need scanning simulation method to help with design and testing of new single and multiple aperture detection schemes.



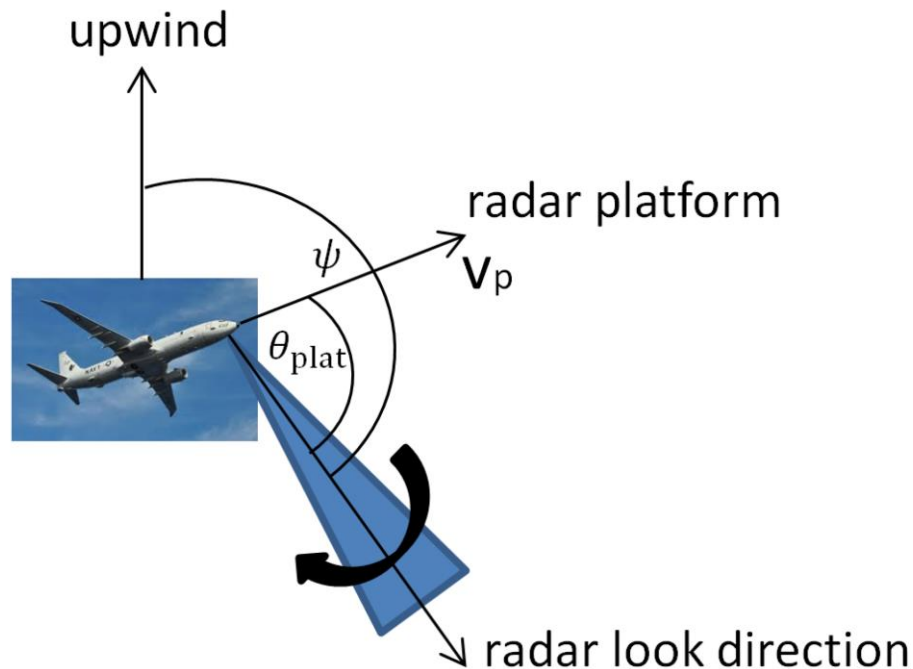
Background

- Significant work over past years to improve sea-clutter modelling techniques.
- Time and spatially varying Doppler spectra model developed by Watts for low grazing angle clutter (2010).
- Found to be consistent with the Ingara medium grazing angle dataset and extended to a bi-modal model for better representing sea-spikes (2014).
- Adapted to work with the Pareto and K+Rayleigh distributions (2014).
- Model was also extended to include multiple phase centres with a single look direction (2014).
- Parameter models developed to enable generic simulation of original and bi-modal Doppler spectrum models (2015).
- Spectrum model is now used to model the sea-clutter observed from a multi-phase centre scanning radar (2016).



Scanning radar scenario

- Scanning radar platform travelling with velocity, v_p .
- Radar look angle is θ_{plat} , angle relative to wind direction is ψ .
- Assume medium grazing angle scenario to utilise existing Doppler parameter models.



Evolving Doppler spectrum model

- Spectrum model is a single Gaussian with normalised underlying mean (texture), τ :

$$G_0(f, \tau, s, \boldsymbol{\Omega}) = \frac{\tau}{\sqrt{2\pi}s(\boldsymbol{\Omega})} \exp \left[-\frac{(f - m_f(\tau, \boldsymbol{\Omega}))^2}{2s(\boldsymbol{\Omega})^2} \right]$$

- Mean Doppler is related to the normalised texture: $m_f(\tau, \boldsymbol{\Omega}) = A(\boldsymbol{\Omega}) + B(\boldsymbol{\Omega})\tau + f_{\text{plat}}$
- Doppler shift due to moving platform: $f_{\text{plat}} = \frac{2v_p}{\lambda} \cos\theta_{\text{plat}}$
- The spectral width, s , is modelled as a Gaussian RV with mean, m_s and std. deviation, σ_s .
- Parameter models required for: A , B , m_s and σ_s .
- Modelled with parameters contained in the set $\boldsymbol{\Omega} = \{\theta_{\text{plat}}, \psi, \theta_{\text{swell}}, \eta, U, H_{1/3}\}$.
- Azimuth two-way beampattern, $A(\theta)$ related to $A(f)$ by $f = \frac{2v_p}{\lambda} \sin\theta$, width is a function of look angle, $\sin(\theta_{\text{plat}})$ – maximum when side looking.
- Can model spreading due to azimuth two-way beampattern through the convolution:

$$G(f, \tau, s, \boldsymbol{\Omega}) = A(f) * G_0(f, \tau, s, \boldsymbol{\Omega})$$

Simulation method

- Simulation method based on the convolution of the desired clutter impulse response g (order N) and the compound Gaussian model $x\sqrt{\tau}$:

$$y(m, \tau) = \frac{1}{\sqrt{2}} \sum_{n=-N/2}^{N/2} g(n, \tau, s, \boldsymbol{\Omega}) x(m+n) \sqrt{\tau(m+n)} \quad (1)$$

- Texture, τ , modelled with a Gamma distribution with Gaussian (spatial) correlation – produces K-distributed clutter model for $y(m, \tau)$.
- Speckle, x , modelled as complex normal RV.
- Discretised impulse response, g found by taking Fourier transform of spectral model where f_r is the PRF:

$$g(n, \tau, s, \boldsymbol{\Omega}) = A \left(\frac{n}{f_r} \right) \exp \left[-j \frac{m_f(\tau, \boldsymbol{\Omega}) 2\pi n}{f_r} \right] \exp \left[-(2\pi s(\boldsymbol{\Omega}) n f_r)^2 \right]$$

- Can be implemented as a sliding window by repeating (1) for each pulse.

Simulation steps

1. Simulate the normalised texture over the entire scan.
 - Only generate realisations for a small number of positions ~ 10 (need approx. constant texture over filter integration time).
 - Distribution shape and spatial decorrelation length determined by user defined inputs in Ω (sea-state, wind and swell direction, grazing angle).
 - Interpolate to the correct time scale (pulse repetition interval).
2. Generate realisations of the speckle RV, x and the spread RV, s (but no scaling or offset).
 - For each pulse, extract correct texture samples from the previous step.
 - Determine model parameters: A , B , m_s and σ_s .
 - Scale and offset the RV s .
 - Form the impulse response g and perform convolution.
 - Add thermal noise with the correct CNR (determined by radar range equation and IRSG-linear mean backscatter model).

Parameter modelling

- Parameter model provides a basis for modelling the statistical parameters in the simulation:
- Used for each polarisation channel independently and contains distinct models for **geometry and sea-state**.
- Geometry variation modelled with Fourier series:
 - $X(\eta, \psi) = a_0\eta^\gamma [1 + a_1\cos\psi + a_2\cos(2\psi) + a_3\cos(\psi - \phi) + a_4\cos(2(\psi - \phi))]$
 - η is the grazing angle, ψ is the azimuth angle,
 - ϕ is the wind swell direction,
 - γ, a_0, \dots, a_4 are the model coefficients.
- Sea-state variation modelled with:
 - $Y = b_0 + b_1\log_{10}(U) + b_2H_{1/3}, \quad (2)$
 - U is the wind speed, $H_{1/3}$ is the significant wave height,
 - b_0, b_1, b_2 are the model coefficients.

Parameter modelling

- To relate these two models, coefficients are altered to be independent of grazing angle by introducing a normalisation factor, η_0 and then redefining previous equation:

$$X(\theta, \psi) = \left(\frac{\eta}{\eta_0}\right)^\gamma [\alpha_0 + \alpha_1 \cos\psi + \alpha_2 \cos(2\psi) + \alpha_3 \cos(\psi - \phi) + \alpha_4 \cos(2(\psi - \phi))]$$

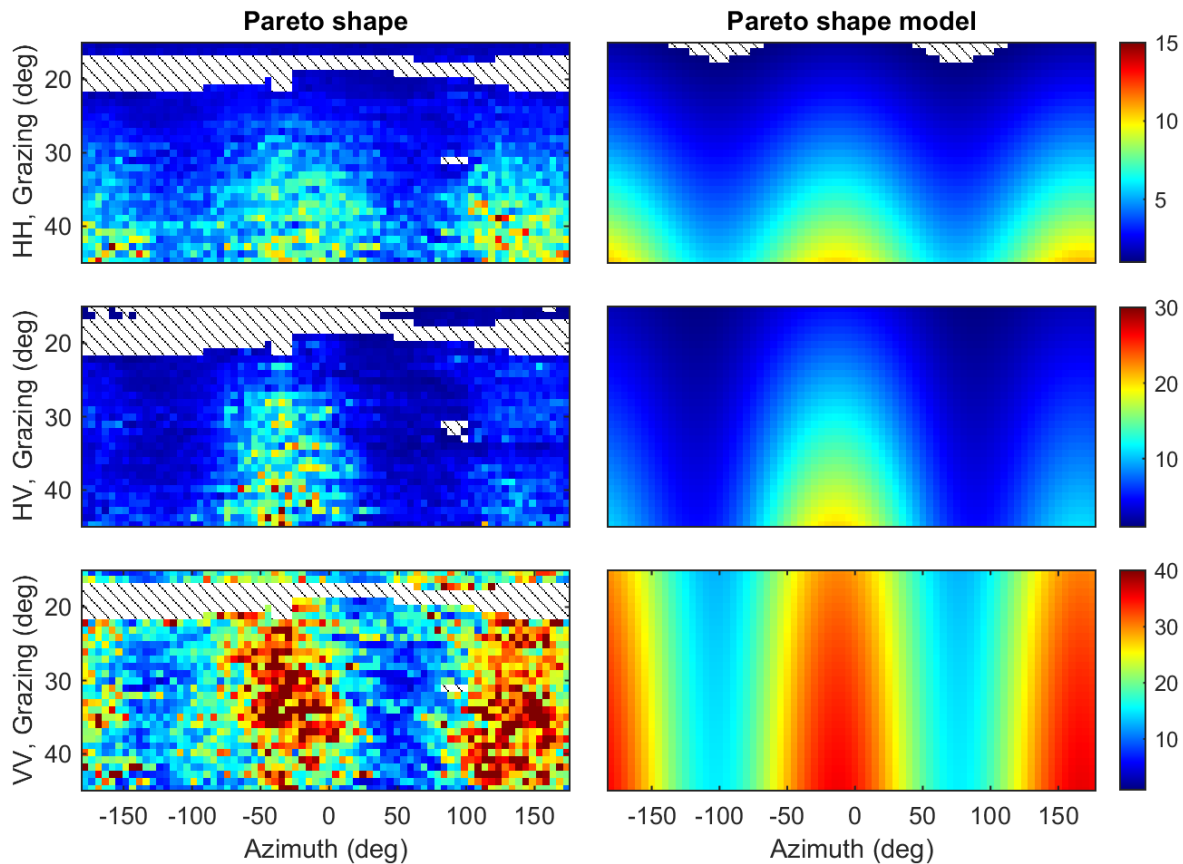
where the new coefficients are related by

$$\alpha_0 = a_0 \eta_0^\gamma, \quad \alpha_1 = a_0 a_1 \eta_0^\gamma, \quad \dots, \quad \alpha_4 = a_0 a_4 \eta_0^\gamma.$$

- The model is then implemented by equating each coefficient $\gamma, \alpha_0, \dots, \alpha_4$ to the model Y in (2).
- Results in 18 coefficients per polarisation.
- Fixed the normalisation factor to 30° .
- Model is used for the shape, ν , spatial decorrelation length, Doppler spectrum parameters A, B, m_s and σ_s .

Example parameter model

- Results show measured Pareto shape / model.
- Some regions where shape is less than 1 - model is not valid.



Modelling sea-spikes

- To improve sea-spike modelling, use bi-modal spectrum model:

$$G_{\text{bi}}(f, \tau, s, \boldsymbol{\Omega}) = (1 - \beta)G_0(f, \tau, s_1, \boldsymbol{\Omega}) + \beta G_0(f, \tau, s_2, \boldsymbol{\Omega})$$

- Gaussian mixture model with ratio β for each component.
- Discrete scatterers due to targets or sea-spikes will cause an extra Doppler spread due to scanning motion of radar:

$$s_{\text{scan}} = \frac{\sqrt{2 \ln 2} \omega_{\text{scan}}}{2\pi \phi_{3\text{dB}}}$$

where $\phi_{3\text{dB}}$ is the azimuth 3dB beamwidth, ω_{scan} is the scan rate.

- Set Doppler spread, $s_1(\boldsymbol{\Omega}) = s(\boldsymbol{\Omega})$, second spread component:

$$s_2(\boldsymbol{\Omega}) = \sqrt{s^2(\boldsymbol{\Omega}) + s_{\text{scan}}^2}$$

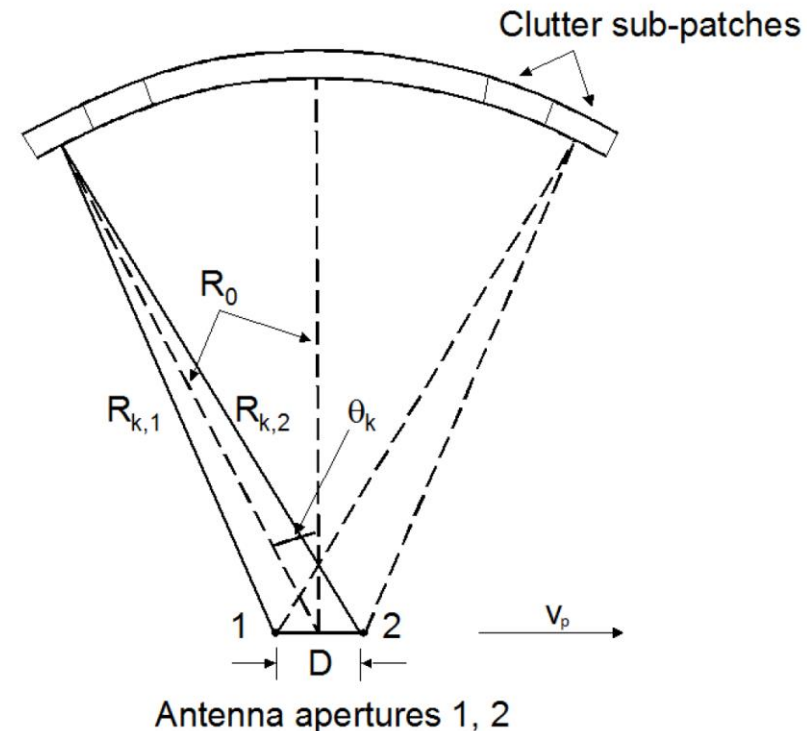
Multiple phase centres

- Require representation of K clutter sub-patches where D is the spacing between phase centres.
- See example for 2 apertures:
- Texture is broken up into K parts:

$$\sum_{k=1}^K \tau_k = \tau$$

where $\tau_k = A(f_k)\tau$ models the return from each part of the beam.

- Can now remove beampattern convolution – i.e just use G_0 or G_{bi} .



Multiple phase centres

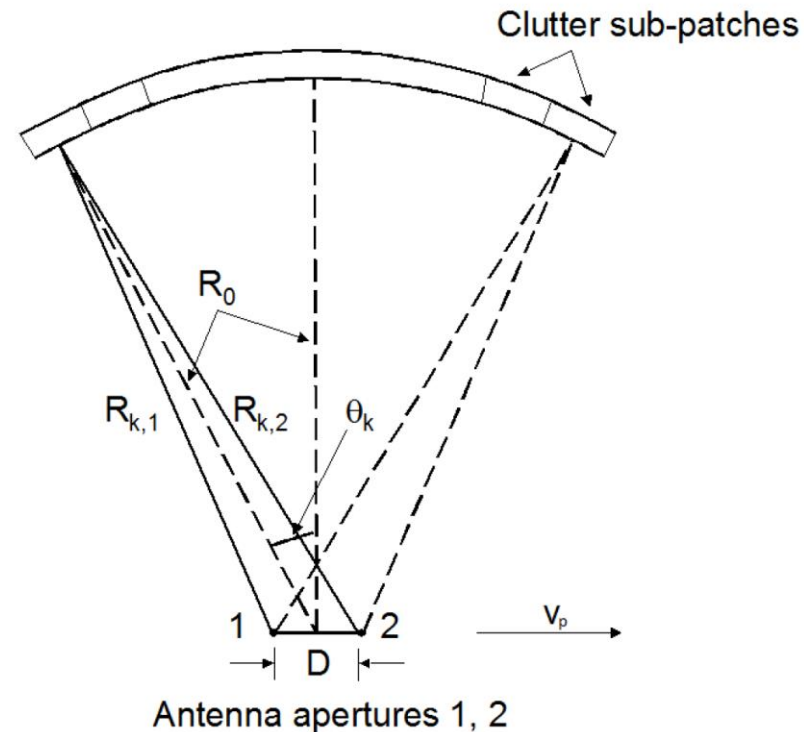
- Need to adjust platform Doppler shift:

$$f_{\text{plat}} = 2v_p / \lambda \cos(\theta_{\text{plat}} - \theta_k).$$

- Overall return from l^{th} sub-aperture:

$$\begin{aligned} \tilde{y}_l(m) &= \sum_{k=1}^K y(m, \tau_k) \exp\left(\frac{j2\pi}{\lambda} (R_{k,l} - R_{k,1})\right) \\ &\approx \sum_{k=1}^K y(m, \tau_k) \exp\left(\frac{j2\pi}{\lambda} D(l-1)\sin\theta_k\right) \end{aligned}$$

- Implementation more computationally intensive due to extra summation.
- Can reduce this by only including those sub-patches where $A(f_k) > \epsilon$, where $\epsilon \approx 10^{-3}$.



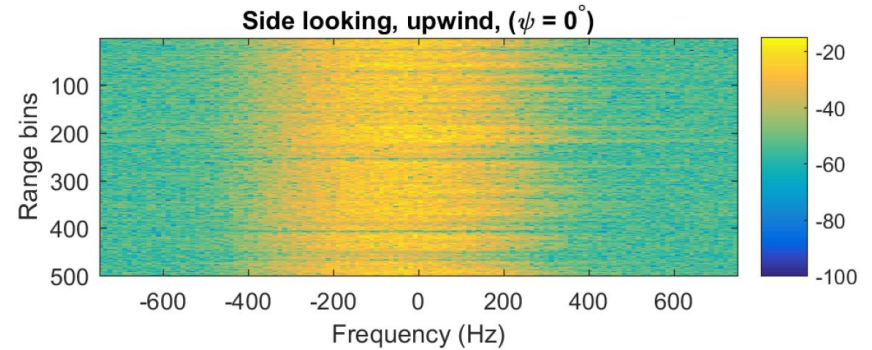
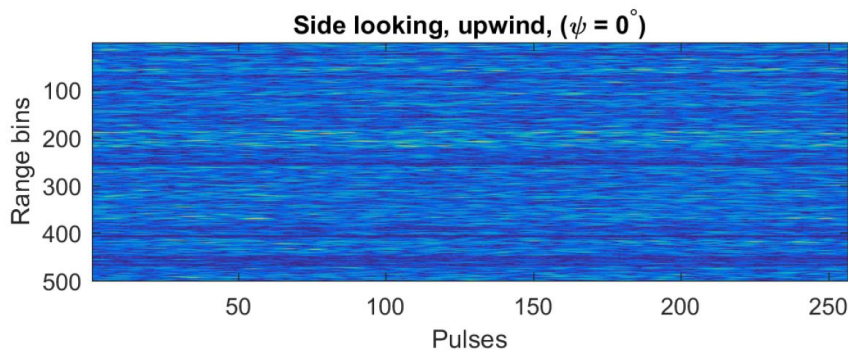
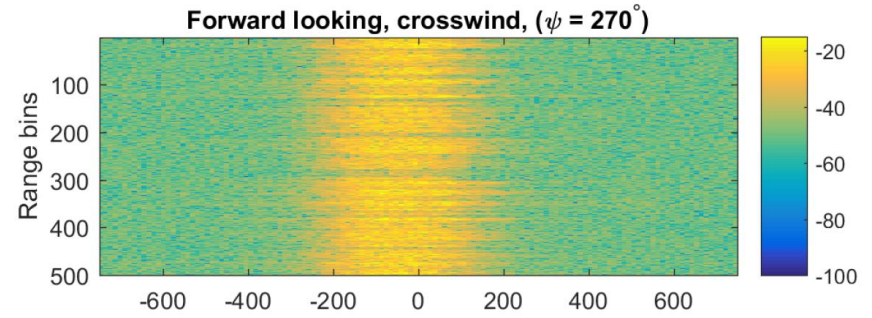
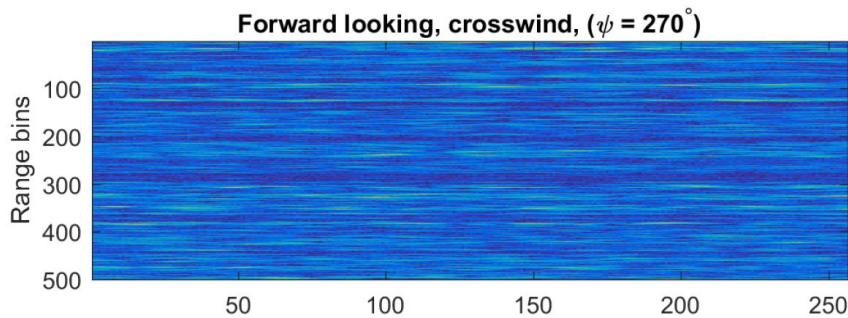
Simulation examples

- Consider the following example with the radar scanning clockwise with the radar platform heading North.
- Wind and swell are both coming from the West.
- Filter order $N=64$.

Radar scan rate	5 rpm
Aircraft speed	80 m/s
Azimuth 3 dB beamwidth (two-way)	3°
Centre frequency	10 GHz
Bandwidth	200 MHz
Pulse repetition frequency	3 KHz
Grazing angle	30°
Sea state	3
No. of pulses a target will be in the beam	300
Amplitude PDF model	K distribution
Spectral model	Bimodal

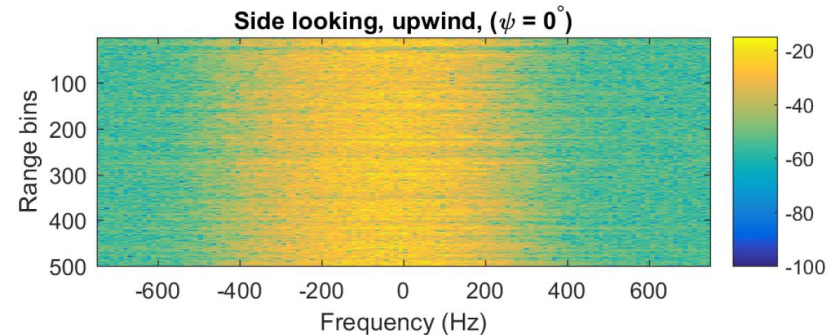
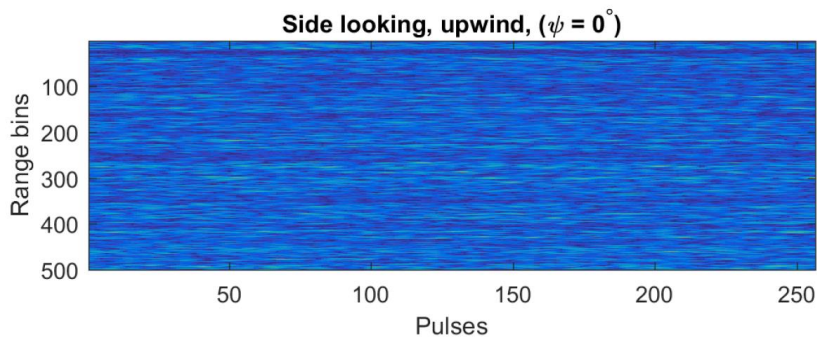
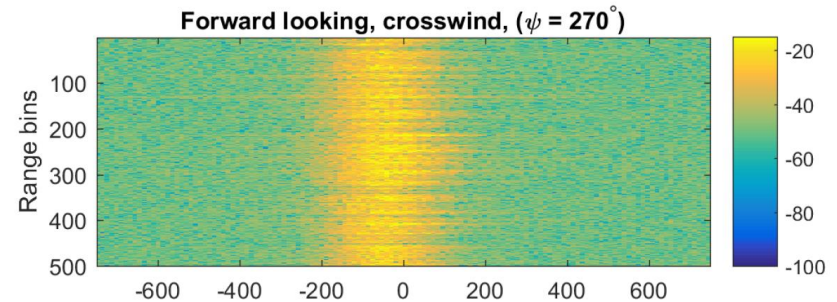
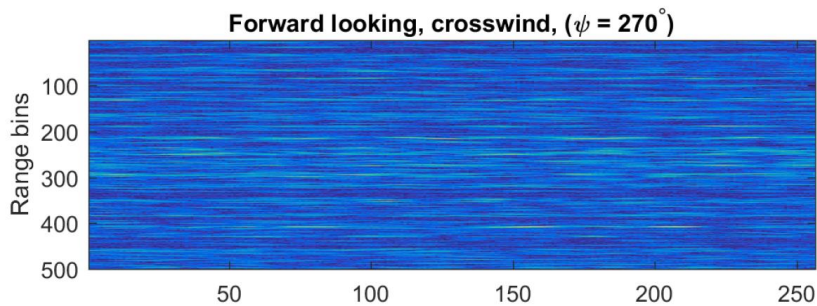
Single aperture results

- Range / time result shown on left for the forward looking (crosswind) and side looking (upwind) cases.
- Doppler spectrum shown on right (after Doppler centroid correction) with greater correlation in the forward looking case (narrower spectrum).



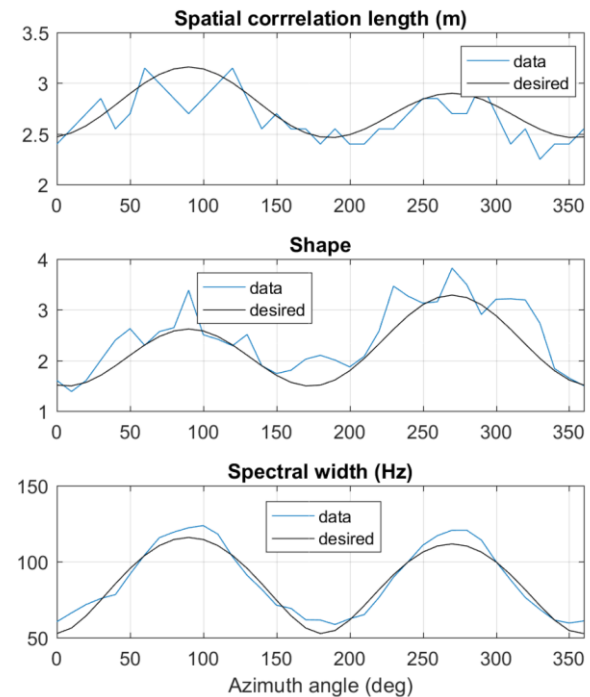
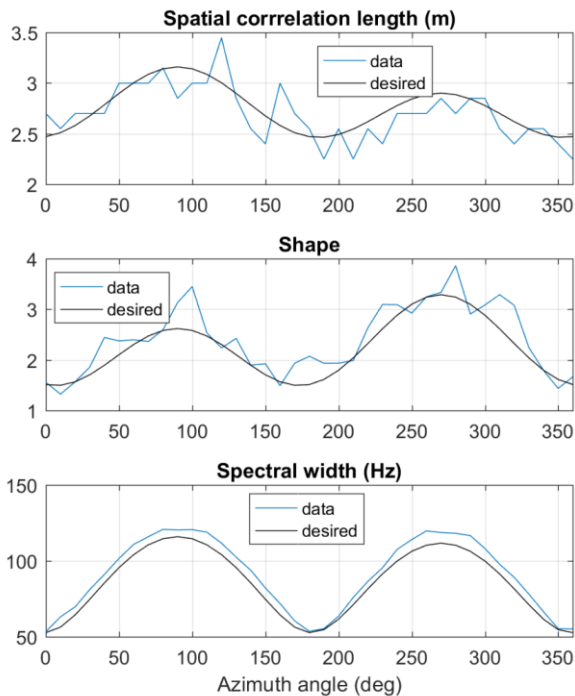
Multi-aperture results

- Multi-aperture model has $D = \frac{\lambda}{2}$ and $L = 10$ spatial channels.
- Forward looking spectrum appears narrower, but on average (over range) the width is correct.



Simulation verification

- Simulation is verified by comparing the estimated spatial correlation, shape and spectral width with the desired / specified values around the orbit.
- All estimated results show a close match with the desired ones.
- Left - single aperture, right – multi-aperture.



Multi-aperture power spectra

- Fourier / optimum power spectrum used to verify the multi-aperture simulation.

$$P_F = \mathbf{v}^H(\theta, f) \hat{\mathbf{R}} \mathbf{v}(\theta, f)$$

$$P_{\text{opt}} = \frac{1}{\mathbf{v}^H(\theta, f) \hat{\mathbf{R}}^{-1} \mathbf{v}(\theta, f)}$$

with steering vectors:

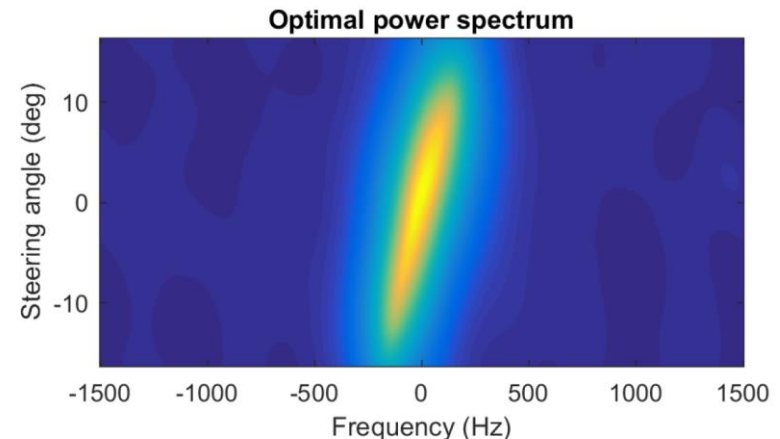
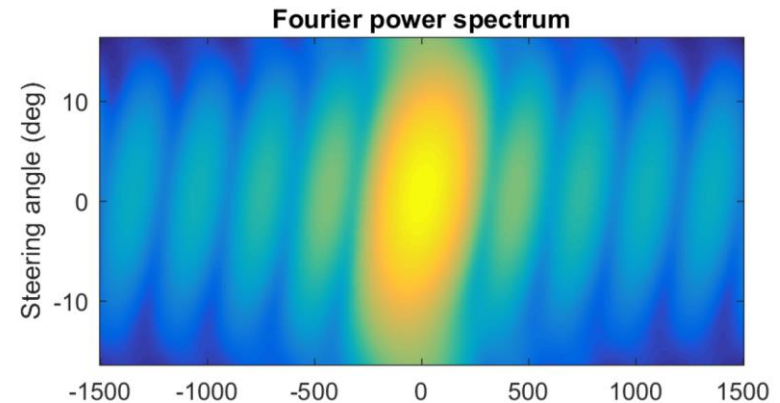
$$\mathbf{v}(\theta, f) = \mathbf{v}_{\text{spat}}(\theta) \otimes \mathbf{v}_{\text{temp}}(f) \in \mathbb{C}^{LC \times 1}$$

containing elements:

$$v_{\text{spat},l}(\theta) = \exp \left[j2\pi(l-1) \frac{d}{\lambda} \sin(\theta) \right]$$

$$v_{\text{temp},c}(f) = \exp \left[j2\pi(c-1) \frac{f}{f_r} \right]$$

Covariance estimated with sample covariance matrix: $\hat{\mathbf{R}} = \sum_{q=1}^{2LC} \mathbf{y}_q \mathbf{y}_q^H$.



Conclusions and future work

- Sea clutter simulation method presented for both single and multi-aperture scanning radars.
- Simulation verified by comparing the estimated shape, spatial decorrelation length and spectral widths with the desired values.
- Multi-aperture simulation also verified by forming the Fourier and optimal spectra.
- Future work will investigate the extension of the parameter models to include:
 - variation of the CNR with range,
 - the impact of varying the coherent processing interval when characterising the model parameters,
 - how to include other range resolutions (beyond 0.75 m).